of the bodies; $\varphi_{\omega,k}$ and $\varphi_{k,\omega}$, mutual irradiance coefficients; Kn, the Knudsen number; L, an operator; λ_k, heat conductivity; $\mu_{k,j,i}$, an arbitrary parameter; P, pressure; Q_k, and Q_ω, the rate of heat flow through the surfaces; Q_b, the flow rate of heat released inside the bodies; q_k, and $q_{k,j}$, the total heat flux and its components; σ, Stefan–Boltzmann constant; T, T_k, and T_ω, temperatures; τ, time.

STATISTICAL PARAMETER CORRECTION FOR MATHEMATICAL MODELS OF HEAT-ENGINEERING SYSTEMS

S. N. Loginov and V. V. Malozemov

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The use of the Kalman-filter equations to calculate parameter corrections for mathematical models of heat-engineering systems is considered.

Recently, discrete (point) models have been used increasingly widely for the calculation and analysis of complex heat-engineering systems. However, the results of such calculations often disagree with experimental data. The sources of possible error may be conveniently divided into three groups [2]: incorrect determination of the functional (structural) design of the system, measurement errors, and errors in the choice of the model parameters.

In the first case, it is necessary to develop a new model. In the last two cases, it is possible to make a statistical estimate of the model parameters using the results of measurements, and so obtain corrected values.

Among the statistical methods used to estimate the parameters of heat-engineering-system models are algorithms based on the equations of the linear optimal Kalman filter; these are of recurrent form and allow the order of the matrices used in the calculations to be considerably reduced. In [1, 4, 5] the filter equations were used in the nonlinear problem of joint estimation of the paremeters and state by linearization of the initial equations in the vicinity of a preliminary estimate. In [2], an estimation problem with initial equations that were linear with respect to the parameters was considered, in the case when the accurate value of the state vector is known. In this formulation, the estimation problem becomes linear and direct solution is possible using the Kalman-filter equations [3]; essentially, it reduces to a recurrent least-squares method.

In the present work, the Kalman-filter equations are used in a parameter-estimation problem for a point model of a heat-engineering system, described by the difference matrix equation

$$\bar{t}(k+1) = A\bar{t}(k) + C\bar{q}(k), \tag{1}$$

or for an individual element

$$t_i(k+1) = t_i(k) + \sum_{j \neq i} \frac{\alpha_{ij}}{c_i}(t_j(k) - t_i(k)) + \frac{q_i(k)}{c_i}. \tag{2}$$

It is assumed that the value

$$\bar{t}^*(k) = \bar{t}(k) + \bar{n}_t(k) \tag{3}$$

is measured, and likewise for

$$\bar{q}^*(k) = \bar{q}(k) + \bar{n}_q(k), \tag{4}$$

where $\bar{n}_t(k)$ and $\bar{n}_q(k)$ are independent random Gaussian series of white-noise type with zero mean and covariance matrices $\mathrm{cov}(\bar{n}_t) = P$, $\mathrm{cov}(\bar{n}_q) = N$. The parameters $1/c_i$ and α_{ij}/c_i are to be estimated. Then, by identity transformations, the equations of state and of observation — Eqs. (1) and (3), respectively — may be reduced to the form

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$$\bar{t}(k+1) = A\bar{t}(k) + C\bar{q}(k) = R(k)\bar{a};\qquad (5)$$

$$\bar{t}^*(k+1) = R(k)\bar{a} + \bar{n}_t(k+1). \qquad (6)$$

The matrix elements of R(k) are the values of the components of the vectors $\bar{t}(k)$, $\bar{q}(k)$ and the known matrix elements of A and C; the vector $\bar{a}$ includes the matrix elements of A and C that are to be estimated. Substituting $\bar{t}(k)$ and $\bar{q}(k)$ from Eqs. (3) and (4) into Eq. (6) gives the result

$$\begin{aligned}\bar{t}^*(k+1) &= A(\bar{t}^*(k) - \bar{n}_t(k)) + C(\bar{q}^*(k) - \bar{n}_q(k)) + \bar{n}_t(k+1)\\ &= R^*(k)\bar{a} - A\bar{n}_t(k) - C\bar{n}_q(k) + \bar{n}_t(k+1).\end{aligned} \qquad (7)$$

The matrix elements of R*(k) are the measured values of the vectors $\bar{t}(k)$ and $\bar{q}(k)$.

Assuming that the parameters of the system are constant, new equations of state and observation for the estimation of the parameters are obtained:

$$\bar{a}(k+2) = \bar{a}(k+1); \qquad (8)$$

$$\bar{t}^*(k+1) = R^*(k)\bar{a}(k+1) + n_a(k+1), \qquad (9)$$

where

$$\bar{n}_a(k+1) = \bar{n}_t(k+1) - A\bar{n}_t(k) - C\bar{n}_q(k). \qquad (10)$$

The new noise $\bar{n}_a(k+1)$ is assumed to be the noise of the (k + 1)-th measurement session. It is evident from Eq. (10) that this is a random Gaussian series with zero mean and covariance matrix

$$\operatorname{cov}(\bar{n}_a) = P + APA^{\mathrm{T}} + CNC^{\mathrm{T}}. \qquad (11)$$

The Kalman-filter equation may be applied directly to the system in Eqs. (8) and (9). Increase in the covariance matrix of the measurement error leads to decrease in the amplification factor of the Kalman filter, as would be expected, since the difference between the predicted and measured values of the vector $\bar{t}(k+1)$ is the result of the additional error due to the inaccuracy of the measurements at time k.

The results obtained allow the Kalman-filter equation to be used successfully in parameter estimation for mathematical models of heat-engineering systems described by equations of the type in Eq. (1), when the measurement results are distorted by Gaussian noise with zero mean.

As an example, consider the heat-engineering system described by the scalar equation

$$\frac{dT}{d\tau} = \frac{1}{c}[\alpha(T_0 - T) + Q\sin(\beta\tau)]. \qquad (12)$$

It is required to correct the parameter 1/c, the standard value of which is 0.0713°K/J, while to a priori value is 0.0144°K/J with dispersion 10^3(°K/J)^2. The measurement value is taken from the analytic solution of Eq. (12) and then distorted by Gaussian noise with zero mean and distortion 0.1(°K)^2. The value of the other parameters were: $\alpha = 0.502 \cdot 10^{-2}$ W/°K; $T_0 = 293$°K; Q = 10 W; $\beta = 0.278 \cdot 10^{-3}$. The correction was carried out in a time interval of 180 sec. For measurements made with a step of 15 sec, the estimate of 1/c was 0.0710 °K/J, and for 60 sec it was 0.0739°K/J. The dispersion of the measurement noise has a great effect on the accuracy of the estimate.

NOTATION

T, t, temperature; α, heat-transfer coefficient; c, specific heat; Q, q, heat liberation; n, random Gaussian series; P, N, covariance matrices. Indices: T, transposition.

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DYNAMIC METHOD OF MEASURING HEAT FLUXES BY BATTERY HEAT FLOWMETERS USING A KALMAN FILTER

A. S. Gol'tsov, D. F. Simbirskii, and S. V. Kudryashov

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A method is developed for the accelerated determination of stationary heat fluxes by battery heat flowmeters in the dynamic measurement mode.

Serially manufactured battery heat flowmeters [1], which are favorably distinguished by the simplicity of construction, the ease of fabrication, and the high response to the flux being measured, are used extensively in the practice of heat-flux measurement.

However, the thermal inertia of these heat flowmeters in certain cases of practical importance will either distort the results being obtained or increase substantially the total time for performing the experiment. From this viewpoint, methods of improving the dynamical characteristics of heat flowmeters, which are realized during subsequent processing of direct measurement, results by solving the inverse problem of heat conduction which occurs, are of indubitable interest. In particular, the problem of computing their values at the initial sections of the transient characteristic of the heat flowmeter can be posed in the measurement of stationary heat fluxes.

The solution of such a problem is described in [2] for the case of measuring the radiant heat flux by using a calorimetric heat flowmeter. The sensor of such a heat flowmeter is a single capacitance link whose dynamics is described by an ordinary differential equation. The magnitude of the heat flux was hence determined successfully by using the familiar Kalman-filter algorithm [3, 4]. However, direct application of a Kalman filter to the problem of measuring a stationary heat flux by battery heat flowmeters is impossible since their dynamics is described by the partial differential equation of heat conduction. At the same time, the Kalman filter is intended for an optimal estimation (in the sense of the root-mean-square deviation) of the state variables of dynamical systems with lumped parameters.

The algorithm of the Kalman filter is distinguished by its simplicity, is quite adaptable for realization on an electronic computer, takes account of the presence of random errors in the measurements, and processes information recurrently as it comes in. It can be applied to both linear and nonlinear dynamical systems. Moreover, utilization of the Kalman filter in the problem of determining the heat flux permits performance of a practical investigation of questions of the uniqueness and accuracy of the results obtained [2], which is especially important since the inverse heat-conduction problem to be solved is hence incorrectly posed.

In order to apply the Kalman filter to the problem of measuring heat fluxes by battery heat flowmeters, an approximate heat-meter model is proposed in this paper which is described by a system of ordinary differential equations and is obtained by the method of lines [5, 6].

In forming the mathematical model, the sensor of the heat flowmeter was considered as a finite rod, heat insulated along the side surface, and executed as a whole with a galvanic copper—Constantan differential thermal battery [1]. Some junctions of the thermal battery are brought out on the endface surface of the sensor which is fastened to the housing and whose temperature is measured by using a Chromel—Alumel thermocouple. Other junctions are disposed on a plane removed a distance $B = 0.2 \cdot 10^{-3}$ m from the detecting surface of the sensor.

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